CERTAIN QUESTIONS OF AEROTHERMODYNAMICS CONNECTED WITH THE MODEL OF FREE-MOLECULAR FLOW

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Heat exchange in very rarefied gases has been treated in a considerable number of papers. In [1,2] and a number of others, the question of determining the equilibrium temperature of a body (a flat plate or a cylinder) moving with constant velocity is considered. The body is assumed to be a perfect conductor and the total heat flow across its surface is calculated. Then, by virtue of the assumption of perfect conductivity, we avoid the unresolved questions connected with the unequal heating of surface elements which are differently oriented in relation to the velocity of motion. Moreover, as a result of it being a steady problem, we leave aside consideration of the process of establishing the temperature equilibrium, which is well known to be important at great heights.

In the present paper, by making use of the expressions obtained in [3] for the heat transfer, we determine the temperature of a thin body which has small unsteady motions,* in addition to its forward velocity, both in the case of purely convective heat exchange and also in the case of radiative heat transfer. The temperature is determined as a function of time, local angle of attack, velocity, the characteristics of the

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* A similar study can be carried out for the determination of the temperature of the surface of thick convex bodies in steady motion, but instead of Expression (1.1) of the present paper we need to use the corresponding expression for the heat transfer derived, for example, in [1,2].

surface and the thermodynamic parameters of the medium at the height under consideration. We notice the important dependence of the time for establishing temperature equilibrium upon the height. For the assumed model of the interaction of the gas with the surface we find the temperature of the gas in contact with the surface and the temperature of the surface, and indicate the dependence of the temperature discontinuity between the gas and the wall upon the assumed model of interaction. We point out a certain similarity between the gasdynamic stagnation temperature and the equilibrium temperature of a plate in free-molecular flow. This fact, together with the similarity established in [3] between the expression for the excess pressure and the "piston theory" formula used in gasdynamics, may provide a certain basis for the application of the expressions obtained here to the temperature in the field of the mechanics of a continuous medium.

1. Let a body be moving through a highly rarefied gas with a constant velocity V relative to a certain reference system fixed in space (the unperturbed velocity), and let it perform small unsteady motions relative to this unperturbed state. Then, according to the kinetic model of a gas assumed in [3], and under the limitations therein prescribed on the shape and motion of the body, we know that the heat transfer in time dt to the surface element ds is determined by the following expression:

$$\Delta Q = \frac{\varepsilon\rho_\infty}{2}\left\{\frac{2Rc^\circ}{\sqrt{\pi}}\left[T_\infty - T_\infty(1-\alpha)\left(1+\frac{2}{3}\frac{V^\alpha V^\beta g_{\alpha\beta}}{c^{\circ 2}}\right)-\alpha T_w\right]+\right.$$
$$+\frac{c^\circ}{2\sqrt{\pi}}\left[V^\alpha V^\beta g_{\alpha\beta}+2V^\alpha w_{./t}^{\beta} g_{\alpha\beta}\right]+\frac{1}{2}V^\alpha V^\beta g_{\alpha\beta}\left(w_{./t}^{3}-V^\gamma w_{./\gamma}^{3}\right)+$$
$$\left.+\frac{R}{2}\left[5T_\infty-4T_\infty(1-\alpha)\left(1+\frac{2}{3}\frac{V^\alpha V^\beta g_{\alpha\beta}}{c^{\circ 2}}\right)-4\alpha T_w\right]\left(w_{./t}^{3}-V^\gamma w_{./\gamma}^{3}\right)\right\} ds\, dt \qquad (1.1)$$

All tensor quantities are considered in coordinates related to the unperturbed constant velocity in the conventional manner. The following notation has been used: ρ_∞, T_∞ are the density and temperature of the incident medium, c° is the most likely velocity of random motion of the molecules, $w_{./s}^{i}$, $w_{./t}^{i}$ are covariant derivatives of the components of the displacement vector with respect to the coordinates and time, $w_{./3}^{3} = -1$, R is the gas constant, $g_{\alpha\beta}$ is the fundamental metric tensor of the coordinates under consideration, α is the accommodation coefficient, ϵ is the coefficient of diffuse reflection, and T_w is the temperature of the surface.

In determining the heating of the body let us take the following thermomechanical model. We shall assume that the layer of the rigid body next to ds (henceforth this layer will be called the surface of the element or body) has a thickness h sufficiently small so that we can assume that the mass included in the volume $h\, ds$ has a uniform temperature

T_w. The inner surface of the layer will moreover be assumed to be adiabatically insulated. Then into the volume $h\ ds$ under consideration there flows through ds a quantity of heat ΔQ on account of the impacts with the gas molecules. Through the side faces there flows a quantity ΔQ_1 on account of the heat conduction from the parts of the body immediately surrounding $h\ ds$. Through ds there is emitted a quantity of heat ΔQ_2 on account of radiation (it is assumed that the surface radiates as a perfect black body according to the Stefan-Baltzmann law). Accordingly, the quantity of heat ΔQ_3 used up in the heating of the surface element is determined from the condition of heat balance by the following expression:

$$\Delta Q_3 = c\rho_w\, hds\, dT_w, \qquad \Delta Q_3 = \Delta Q \;.\; \Delta Q_1 - \Delta Q_2 \tag{1.2}$$

Here c and ρ_w are the specific heat capacity and the density of the body, respectively.

Let us express ΔQ_1 and ΔQ_2 by means of the parameters of the medium and the body. It is easy to see that

$$\Delta Q_1 = kT_{w/\alpha\beta}\, g^{\alpha\beta}\, hds\, dt \tag{1.3}$$

where k is the coefficient of thermal conductivity of the rigid body, $T_{w/\alpha\beta}\, g^{\alpha\beta}$ is the Laplace operator on the function T_w in the coordinates under consideration

$$\Delta Q_2 = \sigma T_w{}^4\, ds\, dt \tag{1.4}$$

and σ is the Stefan-Boltzmann constant*.

Substituting (1.1), (1.3) and (1.4) into (1.2), we obtain, generally speaking, a nonlinear partial differential equation for the determination of the temperature T_w of the surface as a function of the time t, the coordinates x^i, the properties of the surface, the characteristics of the motion of the body, and the thermodynamic parameters of the gaseous medium at the height under consideration:

$$T_{w/t} = \frac{kT_{w/\alpha\beta}\, g^{\alpha\beta}}{c\rho_w} + \frac{\varepsilon\rho_\infty}{2c\rho_w h}\left\{\frac{2Rc^\circ}{\sqrt{\pi}}\left[T_\infty - T_\infty\left(1 + \frac{2}{3}\,\frac{V^\alpha V^\beta g_{\alpha\beta}}{c^{\circ 2}}\right)(1-\alpha) - \alpha T_w\right] + \right.$$
$$+ \frac{c^\circ}{2\sqrt{\pi}}\left(V^\alpha V^\beta g_{\alpha\beta} + 2V^\alpha w_{\cdot/t}^{\ \beta}\, g_{\alpha\beta}\right) + \frac{1}{2}\left(V^\alpha V^\beta g_{\alpha\beta}\right)\left(w_{\cdot/t}^{\ 3} - V^\gamma w_{\cdot/\gamma}^{\ 3}\right) +$$

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* In the case when the body is not perfectly black, we have to attach to the right-hand side of the equation a factor characterizing the degree of blackness.

$$+\frac{R}{2}\left[5T_\infty-4T_\infty\left(1+\frac{2}{3}\frac{V^\alpha V^\beta g_{\alpha\beta}}{c^{\circ 2}}\right)(1-\alpha)-4\alpha T_w\right](w_{\cdot/t}^{\ 3}-V^\gamma w_{\cdot/\gamma}^{\ 3})\Big\}-\frac{\sigma T_w^{\ 4}}{c\rho_w h} \tag{1.5}$$

Questions related to the heat conduction along the surface will not be considered in this paper; as a result, the first term on the right-hand side of Equation (1.5) is neglected. Accordingly, we exclude from consideration the thermal interaction of the parts of the body with each other. Then Equation (1.5) is transformed into an ordinary differential equation with respect to the unknown function T_w, where the time t is the independent variable, and the coordinates x^i are parameters. The displacement vector $\mathbf{w}(x^i, t)$, characterizing the unsteady motion of the surface, is assumed to be a given function of the coordinates and time. Equation (1.5) can be put in the form

$$\frac{dT}{dt}=B_1-B_2T+B_3+B_4-B_5T^4 \tag{1.6}$$

Here we have introduced the following notation:

$$T=T_w/T_\infty,\qquad V^\alpha V^\beta g_{\alpha\beta}/c^{\circ 2}=S^2,\quad \frac{w_{\cdot/t}^{\ 3}-V^\alpha w_{\cdot/\alpha}^{\ 3}}{c^\circ}=S_n,\qquad \frac{\varepsilon\rho_\infty Rc^\circ}{c\rho_w h\sqrt{\pi}}=a_1 \tag{1.7}$$

$$B_1=a_1\left[1-(1-\alpha)\left(1+\frac{2}{3}S^2\right)+\frac{S^2}{2}\right],\qquad B_2=a_1\alpha\,[1+\sqrt{\pi}S_n],\qquad B_3=a_1V^\alpha w_{\cdot/t}^{\beta}g_{\alpha\beta}/c^{\circ 2}$$

$$B_4=a_1\sqrt{\pi}S_n\left[\frac{1}{2}S^2+1.25-(1-\alpha)\left(1+\frac{2}{3}S^2\right)\right],\quad B_5=\frac{\sigma T_\infty^{\ 3}}{\rho_w ch}$$

Let us examine in some very simple particular problems the characteristic singularities of the phenomena which may arise in the assumed models of gas and body (we have in mind the kinetic model of a gas and the thermomechanical model of the body).

2. Let us consider the case of purely convective heat transfer (radiation absent). Equation (1.6) becomes linear

$$\frac{dT}{dt}=B_1-B_2T+B_3+B_4 \tag{2.1}$$

and its general solution is given by the formula

$$T=\exp\left(-\int_0^t B_2dt\right)\left[T_{00}+\int_0^t(B_1+B_3+B_4)\exp\left(\int_0^x B_2dx\right)dt\qquad\left(T_{00}=\frac{T_{w_0}}{T}\right) \tag{2.2}$$

where T_{00} is the dimensionless initial temperature of the element of the surface.

Let the body perform only the unperturbed motion, then the B_i do not depend on time, and the general solution in this case with $B_2\neq 0$ has the form*

$$T = T_{00}\exp(-B_2 t) + [1 - \exp(-B_2 t)]\frac{(B_1 + B_3 + B_4)}{B_2}$$

or, substituting for the B_i their expressions, introducing the local angle of attack β and replacing S by its value

$$S = \sqrt{1/2\varkappa}\, M$$

we obtain

$$T = T_{00}\exp[-a_1\alpha t(1 + \sqrt{\varkappa\pi/2}M\beta)] + \frac{1}{\alpha}\left\{1 + (1-\alpha)\left(1 + \frac{\varkappa M^2}{3}\right) + \frac{\varkappa M^2}{4} + \frac{\sqrt{\varkappa\pi/2}M\beta}{4(1+\sqrt{\varkappa\pi/2}M\beta)}\right\}\{1 - \exp[-a_1\alpha t(1 + \sqrt{\varkappa\pi/2}M\beta)]\} \qquad (2.3)$$

Hence, when $t \to \infty$ we find an asymptotically attained dimensionless equilibrium temperature

$$T_e = \frac{1}{\alpha}\left[1 + (1-\alpha)\left(1 + \frac{\varkappa M^2}{3}\right) + \frac{\varkappa M^2}{4} + \frac{\sqrt{\varkappa\pi/2}M\beta}{4(1+\sqrt{\varkappa\pi/2}M\beta)}\right] \qquad (2.4)$$

On a plate, moving with zero angle of attack, with $a = 1$ the equilibrium temperature becomes

$$T_e = 1 + \frac{\varkappa M^2}{4} \qquad (2.5)$$

According to the restrictions postulated in [3] on the shape and motion of the body ($S_n \ll 1$), the singularities in Expressions (2.3), (2.4) occur outside the limits of applicability of the expression for the heat transfer in the form (1.1) (the appearance of the singularity is a result of introducing the linearization in [3]). The presence of the singularity may serve as a certain indication of the limits of applicable values of $M\beta$.

Let us consider the steady temperature as a function of the local angle of attack. It is easy to see that the temperature of the parts of the surface turned towards the stream are higher than the temperature

* The case $B_2 = 0$, according to the meaning of the quantities determining B_2 (1.7), can occur either when $a = 0$ or when $\epsilon = 0$. The first case corresponds to heat transfer being independent of the temperature of the surface, and it leads, as is apparent from (2.2), to a linear increase with time of the temperature of the surface. In the case $\epsilon = 0$ the heat transfer to the surface is zero and for the whole duration of the motion the temperature remains equal to its initial value.

attained on the flat plate. Let us find the difference in the equilibrium temperatures ΔT_e for two elements of the surface inclined at angles β and $-\beta$ with respect to the direction of the velocity of the incident stream:

$$\Delta T_e = \frac{M\beta}{2\alpha}\sqrt{\frac{\varkappa\pi}{2}}\,\frac{1}{1-\varkappa\pi/2\ M^2\beta^2} \tag{2.6}$$

Calculations carried out for a number of values of M and β show that the difference of temperature at the points of the surface under consideration consists, as a rule, of a small percentage of the corresponding temperature of the flat plate at zero angle of attack, at least in the region of permissible values of $M\beta$ (obviously, with increase in M the permissible values of the angle of attack β diminish). However, it can be shown that allowance for this difference of temperature is important in the determination of thermal stresses arising in structures.

In [1,2] only the equilibrium temperature is determined, which does not depend on the height, as is clear from (2.4). It must be remarked, however, as calculations from Formula (3) show, that at the greatest heights ($\approx$ 100 km and higher) the process of establishing the equilibrium temperature occurs very slowly. This can be seen in the table and Fig.1, where the broken curves show the variation of the temperature T_1 of the body without allowance for radiation, and the full curves show the temperature T_2 of the body with allowance for radiation. So, for example, in the motion of a body at zero angle of attack at heights of 150-200 km with dimensionless velocity $S = 20$ for a period of 2 1/2 hours the temperature of the surface remains practically equal to the initial temperature. Notwithstanding the fact that in the absence of radiation the equilibrium temperature does not depend on height, the time taken to establish this temperature is considerably influenced by the height.

Let us consider the expression for the equilibrium temperature set up on a flat plate (2.5). From boundary-layer theory it is known [4] that in the flow of a steady stream of compressible gas with Prandtl number equal to unity past a plate, with the so-called "adiabatic wall" as the temperature boundary condition (it is assumed that the temperature of the gas at the surface of the plate is equal to the temperature of the plate), there exists an integral of the equation of energy, the Stodola-Crocco integral, which may be written for dimensionless temperatures in the form

$$T_e^* = 1 + \frac{1}{2}(\varkappa - 1)M^2 \tag{2.7}$$

It is not difficult to show the similarity of the equilibrium temperature (2.5) obtained in the case under consideration with the Stodola-Crocco integral. Comparing (2.5) and (2.7), we can see that when $\alpha = 1$

these expressions differ only by the factor involving κ, and they coincide when $\kappa = 2$. When $\kappa < 2$ the equilibrium temperature (2.6), established on the flat plate with free-molecular flow, is higher than the corresponding stagnation temperature (2.7) of continuous flow, which is found in accordance with the results of [2], but not higher than the kinetic temperature* $T_k = 1 + \kappa M^2/3$ of the free-molecular flow with mass velocity V.

In the general case $\alpha \neq 0$ the equilibrium temperature (2.4) established on the surface depends upon the magnitude of α; $\alpha = 0$ is a special case, when the body receives heat but does not give it up, as a result of which the temperature increases without limit and, as is evident from (2.2), there does not exist a steady bounded solution of the problem.

The boundary-layer condition on the surface temperature (equality of the temperature of the surface and of the gas in contact with the surface) is not the only one applicable to the stated problem, and it cannot always apply. Thus, the assumed model of interaction of the molecules of the gas with the surface already determines the value of the temperature of the gas in contact with the wall, which is different from the equilibrium temperature T_e established on the surface itself (2.4). The dimensionless temperature T° of the gas in contact with the surface of the body, obtained by means of the boundary value of the distribution function ([3], Formula (1.1)), is given by

$$T^\circ = \alpha\varepsilon T + \left[\frac{2-\varepsilon}{2}\left(1 + \frac{\varkappa M^2}{3}\right)\left(1 + \sqrt{\frac{\varkappa}{2\pi}}\, M\beta\right)\right] + \\ + \frac{\varepsilon}{2}\left[\left(1 + \frac{\varkappa M^2}{3}\right)(1-\alpha) - \alpha T\right] \tag{2.8}$$

In the case of the commonly accepted value of the accommodation coefficient $\alpha = 1$, Expression (2.7) has the form

$$T^\circ = \varepsilon T + \frac{2-\varepsilon}{2}\left[\left(1 + \frac{\varkappa M^2}{3}\right)\left(1 + \sqrt{\frac{\varkappa}{2\pi}}\, M\beta\right)\right] - \frac{\varepsilon T}{2} \tag{2.9}$$

Accordingly, the model is such that the temperature of the gas in contact with the wall depends upon the kinetic temperature of the gas,

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* The kinetic temperature of a gas is found [5] from the relation

$$\frac{3}{2} RT_k = \int_\Omega c^i c_i f d\mathbf{c} \Big/ \int_\Omega f d\mathbf{c}$$

(Ω is the region of integration, $0 < |\mathbf{c}| < \infty$) in the given case f is the Maxwell distribution with mass velocity $\mathbf{V}$.

the temperature of the wall, its shape and the nature of the interaction of the molecules with the surface.

Let us consider a particular case of the interaction of the molecules with the surface. With pure specular reflection ($\epsilon = 0$) the temperature of the gas in contact with the wall does not depend on the temperature of the wall

$$T^\circ = \left(1 + \frac{1}{3}\varkappa M^2\right)\left(1 + \sqrt{\frac{\varkappa}{2\pi}}\, M\beta\right)$$

and in the case of a flat plate is equal to the kinetic temperature of the gas. In the case of pure diffusive reflection ($\epsilon = 1$)

$$T^\circ = T + \frac{1}{2}\left[\left(1 + \frac{1}{3}\varkappa M^2\right)\left(1 + \sqrt{\frac{\varkappa}{2\pi}}\, M\beta\right) - T\right] \qquad (2.10)$$

and for a flat plate, moving at zero angle of attack, we obtain (with $a = 1$)

$$T^\circ = T + \frac{1}{2}\left(1 + \frac{1}{3}\varkappa M^2 - T\right) \qquad (2.11)$$

From (2.11) we can see that $T^\circ < T$ if the surface possesses a temperature higher than the kinetic temperature of the gas, and vice versa. Accordingly, generally speaking, at the contact surface of the gas with the surface (within the assumptions of the accepted model of the gas, the surface and their interaction) there exists a discontinuity of temperature, and only in the special case when the flat plate warms up to the kinetic temperature of the stream does the temperature of the gas in contact with the flat plate equal the temperature of the plate itself. Moreover, this temperature is different from the equilibrium temperature, set up in the plate as a result of only convective heat transfer.

Expression (2.8) for the temperature of the gas in contact with the wall can be used as a boundary condition in the solution of temperature problems in the gas.

3. Let us consider heat transfer with radiation. In the case of unperturbed motion of the surface the B_i do not vary with time, the variables in the equation (1.6) are separated, and its solution can be obtained in quadratures

$$t = \int_{T_\infty}^{T} \frac{dT}{B_1 - B_2 T + B_3 + B_4 - B_5 T^4} \qquad (3.1)$$

Calculations which have been carried out show (Fig. 1 and the table) that at the greatest heights (150 km and higher) radiation plays a fundamental part in heat transfer from the surface to the medium. If we take for the initial temperature the temperature of the surrounding

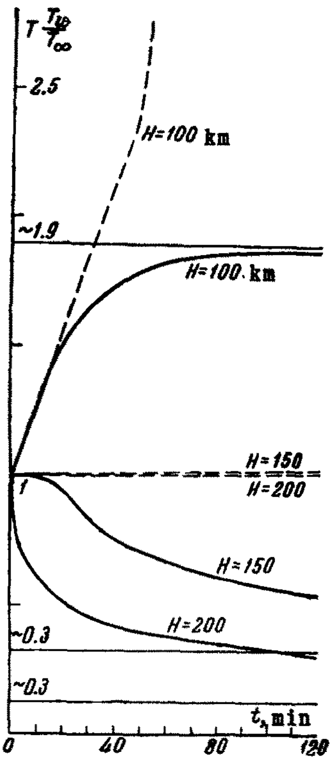

Fig. 1.

Dependence of dimensionless temperature on time with $\alpha = 1$, $\epsilon = 0$, $S = 20$, $c = 0.12$ cal.cm g^{-1} deg^{-1}, $\rho_w = 7.9$ g cm^{-3}, $h = 0.5$ cm.

H, ρ_∞ T_∞	t	T_1	T_2
$H=100$ km, $\rho_\infty=0.829\cdot10^{-9}$ g cm^{-3}, $T_\infty=237°K$	10	1.28	1.27594
	20	1.6	1.51412
	30	1.8	1.69201
	40	2.196	1.80242
	50	2.396	1.8587
	60	2.792	
	∞	201	1.889
$H=150$ km, $\rho_\infty=0.34\cdot10^{-11}$ g cm^{-3}, $T_\infty=418°K$	10	1.002	0.87623
	20	1.004	0.80394
	30	1.006	0.75319
	40		0.71447
	50	1.008	0.68343
	60	1.00995	0.65773
	70	1.012	0.6359
	80	1.014	0.61703
	90	1.0146	0.60049
	100	1.016	0.58572
	110	1.018	0.57269
	120	1.02	0.56058
	130	1.022	0.55007
	140	1.024	0.54022
	150	1.026	
	∞	201	0.328
$H=200$ km, $\rho_\infty=0.166\cdot10^{-12}$ g cm^{-3}, $T_\infty=647°K$	10	1.00091	0.53505
	20	1.002	0.49705
	30	1.0028	0.46875
	40	1.0038	0.4464
	50	1.0048	0.42303
	60	1.0058	0.41252
	70	1.0068	0.39915
	80	1.0076	0.38745
	90	1.0086	0.35997
	100	1.0096	0.33954
	110	1.0106	
	120	1.0116	
	130	1.0124	
	140	1.0134	
	150	1.0144	
	∞	201	0.121

medium, then, as is evident from Fig. 1, the equilibrium temperature in the presence of radiation heat transfer, beginning at a height of 150 km, is lower than the temperature of the surrounding medium and much lower than the equilibrium temperature found in a calculation taking account only of purely convective heat transfer.

And here, as also in the preceding case (Section 2), it is necessary to pay attention to the duration of the process of establishing the equilibrium temperature. The variation of the temperature with time for different heights was obtained by numerical integration of Equation (1.6), the thermodynamic characteristics of the medium at different heights being obtained from [6,7]. In the calculations the surface was taken to have the following characteristics: $\epsilon = 1$, $a = 1$, $c = 0.12$ cal.g^{-1} cm^{-3} deg^{-1}, $\rho_w = 7.9$ g cm^{-3}, $h = 0.5$ cm.

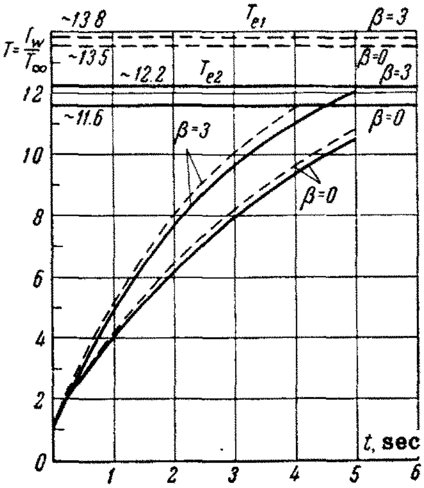

Fig. 2.

4. The results obtained in this paper, generally speaking, are valid only for great heights in the region of free-molecular flow. We have already noticed the analogy established in [3] between the pressure in free-molecular flow and the flow of an ideal compressible fluid. Comparison of Formulas (2.7) and (2.5) of the present paper allows us to perceive a similar analogy with regard to temperatures. Since in the mechanics of a continuous medium we have still not discovered a simple relation between the temperature of the surface or the heat intake and the local angle of attack in unsteady motion, there is some interest, in view of the specified analogy, in assuming (up to the present time, we have failed to obtain the rigorously proved dependence of temperature and heat intake on the local angle of attack) as a hypothesis for the gas-dynamic calculations the expression obtained in form (1.1) for the heat intake and all the consequences arising therefrom.

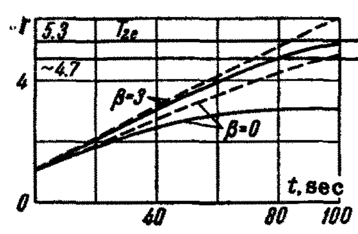

Fig. 3.

Computations were carried out for the determination of temperature at heights 20-50 km, both in the case of purely conductive heat transfer, and also for heat transfer with radiation when $S = 5$. The results show that with decrease of height the part played by radiation in the overall heat balance decreases (we must not, however, forget that in this problem we have considered only the radiation of the surface itself, and have not taken into account the influence of radiation of the gas). In Figs. 2 and 3 are shown the variation with time of the temperature of the body T_1 without allowing for radiation, and T_2 with allowance for radiation. Figure 2 shows the process of establishing the equilibrium temperature at a height of 20 km, and Fig. 3 at a height of 50 km. T_{e1} and T_{e2} denote the equilibrium temperatures in the absence and presence of radiation, respectively. From the graphs it is clear that as the height decreases the process of establishing the equilibrium temperature proceeds significantly faster. In the graphs we can also see the influence of the angle of attack on the process of establishing the equilibrium temperature.

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